LONDON SOUTH BANK UNIVERSITY

unit guide

Introductory Mathematics

UNIT ECM_1_185

Faculty
of
Engineering, Science and Built Environment

2008 - 2009

become what you want to be

Table of contents

1. UNIT DETAILS

Title	**Introductory Engineering Mathematics**
Number/Value/Level	ECM-1-801/1
Pre-requisites	GCSE maths or equivalent
Courses	B.Eng. (Honours) in Electrical and Electronic Engineering, Built Environment Engineering, Chemical Engineering, Mechanical Engineering
Status	The unit has been validated July 2002.
Time allocation	150 hrs: 44 hrs lecture/22 hrs tutorial/84 hrs study time
Co-ordinators	Prof Larissa Fradkin, T818, ext. 7560, e-mail: fradkil@lsbu.ac.uk Prof Lawrence Dunne, T800, ext: 7944, e-mail: dunnel@lsbu.ac.uk Dr Geoff Goss, T810, ext. 7625, e-mail: gossgal@lsbu.ac.uk

Information for Students with Disabilities / Dyslexia

The Learning Support Unit provides services for students with disabilities and dyslexia at both the pre-entry stage and while studying at London South Bank University. Advice and support is available for all students with a Disability/ Dyslexia to enable you to manage your specific academic and practical needs. This includes:

* An initial screening and full assessment if you think you may have dyslexia
* One to one advice and guidance
* Arrangements for examinations, assessments, and on course provision, (i.e. extra time in exams, possible extensions)
* Advice and training for staff
* Disability and dyslexia workshops
* One to one tutorials
* Student focus groups
* Disability access across campus
* Technical support and access
* Support worker service

If you are a student with a disability or dyslexia or think you might need to be assessed for dyslexia, please contact the LSU as soon as possible. The sooner any arrangements you need can be made, the better equipped you will be to succeed on your course.

Learning Support Unit, Caxton House: Tel: 020 7815 6400; Web: www.lsbu.ac.uk/caxton

2. SHORT DESCRIPTION

This unit guide is intended to provide a general idea of the teaching content and assessment criteria for the unit entitled Introductory Engineering Mathematics. This is a long thin unit, which is taught in Semesters one and two. It is delivered as a two-hour lecture every week. Tutorials take place once a week.

The unit covers the mathematical and theoretical foundations for the BEng scheme. Together, they allow students to assimilate related material over the whole academic year.

3. AIMS OF THE UNIT

The unit provides students with the necessary mathematical tools and methods needed in all other engineering programmes in the scheme, such as algebra, complex numbers and calculus. It will also give students transferable skills, such as study skills, participating in technical discussion and recognising a familiar pattern in an unfamiliar picture.

4. LEARNING OUTCOMES

4.1 Knowledge and understanding

At the end of study:

1. You should understand and be able to perform simple algebraic manipulations, including operations on complex numbers.

2. You should understand the basic calculus and be able to sketch, differentiate and integrate functions of one real variable

3. You should be able to understand and be able to perform operations on vectors and matrices and solve systems of linear algebraic equations.

4.2 Intellectual skills

You should learn to use the computer resources learning and aquire problem-solving skills, such as learning the jargon, learning the basics; recognising familiar patterns in unfamiliar pictures; prioritising; learning to participate in technical discussion

4.3 Transferrable skills

You should learn to study regularly and behave in a professional manner: be punctual and considerate to others to assure smooth work of a team.

7. ASSESSMENT

The unit has **two** components of assessment (remember that assessment components are what will appear on your transcript and that you have to get a minimum mark of 30% for each component as well as an overall unit mark of 40 % to pass the unit). The components are:

1. A three hour Maths **Exam** at the end of Semester 2, which will contribute 50% of the final unit mark.

2. Continuous **in-course assessment**, which will contribute 50% of the final unit mark. The in-course assessment consists of **three** sub-components:

i) A three hour **Phase Test** taken at the end of Semester 1, which will be 60% of the in-course assessment (i.e. 30% of the final unit mark). This test will be normally conducted during the exam week, after Semester 1. This test is to check that students have mastered the basic techniques covered in Semester 1, as a foundation for the Semester 2 work.

ii) A coursework mark for the **Semester 1,** arrived at by examining the **Logbook** submitted to the **LECTURER via the Faculty Office** immediately after the Phase Test. It will take into account

- Tutorial Attendance
- Homework
- Quality of logbook
- Revision notes

as evidenced by your Logbook and will be 20% of the in course assessment (i.e. 10% of the final unit mark)

iii) A coursework mark for the **Semester 2,** arrived at by examining the **Logbook** submitted to the **LECTURER via the Faculty Office** immediately after the Exam. Again, it will take into account

- Tutorial and Workshop Attendance
- Homework
- Quality of logbook

- Revision notes

as evidenced by you Logbook and will be 20% of the in course assessment (i.e. 10% of the final unit mark).

BOTH LOGBOOKS SHOULD BE RETRIEVED WITHIN A MONTH AFTER

SUBMISSION. INFORMATION ON HOW TO RETRIEVE LOGBOOKS WILL BE POSTED ON THE BLACKBOARD.

6. FEEDBACK

Each week Solutions to Homework Problems will be discussed in Tutorials. They will be published on the Blackboard the week after that.

Solutions to the Winter Phase Test will be discussed in class in Week 1 of Semester 2. The list of Common Mistakes will be published on the Blackboard. Students who don't come up to standard in the winter Phase Test (that is do not score at least 40%) will be given a chance to rework it then, using this information, in Week 2 f of Semester 2 (with the repeat mark capped at 40%). **THEREFORE**, **DO NOT MISS THE FIRST WEEK OF SEMESTER 2!**

The Faculty Office will send of letters explaining results according to the Faculty deadlines.

7. INTRODUCTION TO STUDYING THE UNIT

7.1 Overview of the main content

Topic	SI: Study Weeks
Introduction and revision of algebra	1
Revision of algebra: Solving Simple Eqns (ctd)	2
Functions of real variable: Diagrammatic & graphical representation	3
Revision of algebra Functions of real variable: Polynomials	4
Functions of real variable: exponentials, log and trig functions	5
Functions of real variable: Inverse trig & hyperbolic fns	6
Sequences, Series and Limits	7
Differentiation	8
Differentiation rules & Decision tree	9
Integration of functions (Definite integrals)	10
Integration of functions (Indefinite integrals)	11
REVISION	12
Topic	**SII: Study Weeks**
Complex numbers	1
Complex numbers (ctd)	2
Complex Numbers (ctd)	3
Sketching and Using Function Graphs; Sketching Functions Using Simple Transformations	4
Sketching Functions Using Several Simple Transformations; Pointwise Ops and Analysis	5
Sketching Functions Using Analysis(ctd)	6
Sketching Rational Functions	7
Taylor and Maclauren Series	8
Integration Methods	9
Applications of Integration	10
Revision of Differentiation and Integration	11
REVISION	12

Notes:

a) This programme allows lectures to finish in week 11 of each Sememster to give one week for revision and also to provide for possible over-run on the lectures.

b) The above calendar should serve as a relatively accurate guideline, which is to say that some minor alterations may take place in order, achieve the best sequence of lectures.

The calendar shown above offers the general teaching framework and provides some flexibility to allow diversification in teaching material, if this is deemed suitable.

7.2 Overview of types of classes

Teaching is by twelve two-hour **lectures** and by twelve one or two-hour **tutorial sessions**, depending on whether you are a part or full-time student. All students are asked to put **Lecture Notes** in one notebook and homework and tutorial work into a separate **Logbook**.

The teaching method that is adopted is based on the Harvard System of Socratic Dialogue. The interactive atmosphere enables the students to participate in the learning process, learn to work in a team and provides both students and teachers with an immediate feedback**. If students feel unwilling or unable to participate they are asked to inform the teacher before the class commences.**

Students are expected to discuss the difficulties they have encountered with the teacher during tutorials. Thus, while the lectures are teacher driven the tutorials are student driven.

The TUTORIALS are designed to enable student to use the theory covered and SHOULD NOT BE MISSED. The logbooks will be signed by tutors every week at the end of the tutorials. These signatures will be used as to assess tutorial attendance.

7.3 Importance of student self-managed learning time

All students are encouraged to use the Study Skills Resources Centre to learn how to self-manage learning time.

When it comes to maths, on average you are supposed to spend an hour of self-study for every hour of contact with a math teacher.

In the Teacher Tips folder on the Blackboard site for this unit you will find documents such as Study Skills and Examination Techniques which will explain in detail how to manage your learning time. If you follow the advice given there you will learn much faster and efficiently than otherwise.

To summarise the message given in these documents, every week you are supposed to

- Revise last week's lecture from Lecture Notes (your own and the copy given on the Blackboard) and tutorial work from your Logbook
- Study your Lecture Notes and Lecture Notes on the Blackboard for this week
- As part of your course work put a flow chart of each lecture into your Logbook
- Attempt your homework in your Logbook
- Attend a Lecture and put your Lecture Notes into one notebook and attend Tutorial and put your tutorial notes into your Logbook

MOST STUDENTS WHO FAIL DO SO THROUGH LACK OF STUDY SKILLS OR LACK OF ENGLISH!

Students whose English is not fluent are encouraged to use the Language Laboratory in the LRC.

7.4 Employability

Knowledge of mathematics is important for engineers and employers actively seek those who understand basic ideas. The transferable skills acquired this unit revolve around systematic approach to tasks and team work. Both are invaluable in a modern working environment.

8. THE PROGRAMME OF TEACHING

The syllabus is taught in one two-hour slot per week. The teaching atmosphere will be aimed at encouraging discussions and student interaction. The schedule given below should therefore be considered as a useful guide to the lecture content rather than an exact description of the material that will be delivered in the lecture. The schedule shows eleven teaching weeks and a week of revision. The revision session is intended to cover some important topics that are difficult to grasp, and also to offer a buffer period in case some lectures need to over-run.

Semester 1 Week	**Content**	**Refer-ences**
Week 1 (2hrs)	Introduction and revision of Algebra The reason for studying maths are discussed; revision is given of laws and rules of addition, and subtraction. Various types of numbers are reviewed, whole, natural and integers. The concept of inverse operations and number line is introduced. Learning outcomes Be thoroughly familiar with the types of numbers used in engineering, elementary algebraic operations and order of operations. Know the meaning of words **term** and **sum**.	Ref:1-3
Week 2 (2hrs)	Introduction and revision of Algebra Revision is given of laws and rules of multiplication and division. New type – of rational numbers is introduced. The concept of inverse operations and number line are reinforced is introduced. The order of operations is discussed. An algorithm for solving simple equations as based on these concepts is given. Learning outcomes Be thoroughly familiar with the types of numbers used in engineering, elementary algebraic operations and order of operations. Be able to remove brackets, factorise, solve simple equations and add fractions. . Know the meaning of words **factor** and **product**.	Ref:1-3

Week 3 (2 hrs)	Real Functions of One Real Variable Definition of a real function of one real variable and its diagrammatic representation is given; concepts of domain and range are introduced. Elementary operations on function are defined: addition, subtraction, multiplication and division, as well as new operations: composition and inverse function. Learning outcomes Be thoroughly familiar with the concepts of function, domain and range and learn to perform operations on functions. Know the meaning of words **function, argument** and **domain**.	Ref:1-3
Week 4	Special Types of Functions This lecture will introduce polynomials, with the emphasis on their domains and ranges. The concept of the frame of reference and co-ordinates of a point will be revised. The graphs of the above functions will be sketched using tables. Learning outcomes Know how functions can be presented in a graphical form and the ways of sketching a straight line, a parabola and any polynomial using tables	Ref:1-3
Week 5 (2hrs)	Special Types of Functions This lecture will introduce exponential, logarithmic an trigonometric, with the emphasis on their domains and ranges. The concept of the frame of reference and co-ordinates of a point will be revised. The graphs of the above functions will be sketched using tables. Learning outcomes Know how functions can be presented in a graphical form and the ways of sketching exponentials, log and trigonometric functions using tables	Ref:1-3
Week 6 (2hrs)	Special Types of Functions This lecture will introduce inverse trigonometric and hyperbolic functions, with the emphasis on their domains and ranges. The graphs of the above functions will be sketched using tables. Learning outcomes Know how functions can be presented in a graphical form and the ways of sketching introduce inverse trigonometric and hyperbolic functions using tables	Ref:1-6

Week 7 (2hrs)	Sequences , Series and Limits The concepts of sequence, series and limit are introduced; arithmetic and geometrical progressions are defined; the Binomial Theorem is given. Learning outcomes Be familiar with the above concepts.	Ref:1-6
Week 8	Functions (ctd.): Limits, Continuity and Derivatives, The concepts of a limit, continuity and derivative will be introduced. The geometrical interpretation of a derivative as a local slope will be given as well as its mechanical interpretation as a rate of change. Derivatives of a few simple functions will be calculated from the first principles. A table of derivatives of a few special functions will be introduced. The simplest ordinary differential equation y' = y will be considered. Learning outcomes Know what are continuous functions and memorise derivatives of a few special functions. Know the geometrical and mechanical interpretation of a derivative.	Ref:1-6
Week 9 (2hrs)	Differentiation Rules and Decision Tree This lecture will introduce differentiation rules to enable students differentiate sums, products and quotient of functions as well as composite and inverse functions. Learning outcomes Know how to differentiate.	Ref:1-6
Week 10 (2hrs)	Integration The concept of a definite integral will be introduced. Geometrical interoperation of a definite integral as a (signed) area underneath a curve will be given. Learning outcomes Know the basic definition of a definite integral and geometrical interpretation of a definite integral.	Ref:1-6
Week 11 (2hrs)	Integration The concept of an indefinite integrals (antiderivative) will be introduced. A table of indefinite integrals of a few special functions will be introduced Learning outcomes Know how to integrate simple functions.	Ref:1-6
Week 12	Revision	

Semester 2 Week	**Content**	**Refer-ences**
Week 1 (2hrs)	<u>Complex Numbers</u> The concepts of the imaginary unity j and complex number will be introduced. Elementary operations on complex numbers will be defined: addition, subtraction and multiplication by a real number. Both Cartesian and polar representation will be used. <u>Learning outcomes</u> Know how to represent complex numbers. and how to perform elementary operations on them.	Ref: 1-5
Week 2 (2hrs)	<u>Complex Numbers</u> The concepts of the imaginary unity j and complex number will be introduced. Elementary operations on complex numbers will be defined: addition, subtraction and multiplication by a real number. Both Cartesian and polar representation will be used. Euler's formula and thus the exponential representation of the complex number will be introduced. <u>Learning outcomes</u> Know how to represent complex numbers and how to perform elementary operations on them (addition and subtraction).	Ref: 1-5
Week 3 (2 hrs)	<u>Complex numbers (ctd.)</u> Further operations on complex numbers will be defined: Further operations on complex numbers will be defined: multiplication, complex conjugate, and division, taking power (DeMoivre's Theorem) and roots. <u>Learning outcomes</u> Know how multiply and divide complex numbers. Know how take power and root of a complex number.	Ref: 1-5
Week 4 (2hrs)	<u>Sketching and Using Graphs</u> This lecture will introduce simple transformations used in sketching graphs: translation along x- or y-axis, scaling along x- or y-axis, reflection with respect to x- or y-axis; sketching of inverse function. <u>Learning outcomes</u> Know how to sketch graphs using simple transformations.	Ref: 1-3

Week 5 (2hrs)	Sketching Functions (ctd) This lecture will demonstrate how to sketch graphs using several simple transformations; pointwise operations; and how to use derivatives to sketch graphs of differentiable functions: how to find stationary points (where the first derivative is zero), to establish whether they are max, min or neither, to establish regions where the function is increasing or decreasing; Learning outcomes Know how to sketch graphs using several simple transformations, pointwise ops and derivatives and limits.	Ref:1-3
Week 6 (2hrs)	Sketching Rational Functions It will be shown how to use derivatives and limits to sketch rational functions. Learning outcomes Know how to sketch rational functions.	Ref:1-3
Week 7 (2hrs)	The Taylor and Maclauren Series The Taylor and Maclauren series will be introduced. Their applications to function approximation will be discussed. Learning outcomes Know how to find terms of the Taylor and Maclauren series of differentiable functions., and thus approximate differentiable functions.	Ref:1, 2,4,6
Week 8 (2hrs)	Integration Methods This lecture will introduce integration by parts, integration of trig fns and integration of rational functions using partial fractions. Learning outcomes Be familiar with simple integration methods.	Ref: 1-6
Week 9 (2hrs)	Application of Integration (ctd) This lecture will demonstrate how to use calculus to model electrical, translational and rotational systems. Learning outcomes Understand how the engineering systems are described using the basic concepts of calculus.	Ref:1-6

Week 10 (2hrs)	Ordinary Differential Eqns This lecture will introduce ordinary differential equations and initial value problems. It will be shown how to use simple ordinary differential equations y' = ay to describe linear engineering systems, in particular, chemical engineering. Homework will cover all three, electrical, mechanical and chemical applications. Learning outcomes Understand how the engineering systems are described using the basic concepts of calculus.	Ref: 1,2,4-6
Week 11 (2hrs)	Revision of differentiation and Integration	Ref: 1,2,4-6
Week 12 (2hrs)	Revision	

9. LEARNING RESOURCES

9. 1 Core Materials

THE MAIN SOURCE OF INFORMATION IS LECTURE NOTES, SOLUTIONS TO TUTORIAL SHEETS, HANDOUTS AND various other documents put on the webpage for IntroductoryMathematics that you can find on the **BLACKBOARD** (http://www.lsbu.ac.uk/bb). We expect every student to consult the Blackboard every week. **Please attach the Unit Guide, including ALL handouts, at the end of your Lecture Notes.**

Also, students will now have access to the computer package **MATHLETICS** which will allow you to **self-assess**.

On top of that you are advised to buy

Ref. 1 A. Croft and R. Davidson 1999 Mathematics for Engineers: A Modern Interactive Approach
Pearson Education Ltd, 1028 pp.
ISBN 0 130 333 484

This is a comprehensive text on calculus, which deals with an extensive range of topics. Some topics will be explained in lectures in more detail. It should be used mainly as a reference and source of exercises.

9.3 Optional Materials

Ref. 2 A. Croft, R. Davidson and M. Hargreaves 1996 Engineering Mathematics,
Prentice Hall, 3rd edition, 969 pp.
ISBN 0-130-26858

A very solid text for engineering students. A source of many additional exercises.

Ref. 3 A. Jeffrey Mathematics for Engineers and Scientists
Chapman and Hall , 5th edition911 pp.
ISBN 0 412 621509

A slightly more advanced version of the above. Again, a source of additional exercise.

Ref. 3 L. Bostock, and S. Chandler 1994 CORE MATHS FOR A-LEVEL, 2nd ed., Stanley Thornes, 875 pp.
ISBN 0748700676

Many basic concepts are explained well**.**

Ref. 4 A. Jeffrey 1995 Essentials of Engineering Mathematics,
Chapman and Hall, 825 pp.
ISBN 0-412-39680-7

You might find this book helpful.

Ref. 5 K. A. Stroud 1995 Engineering Mathematics,
The Bath Press, 1032 pp.
ISBN 0-333-62-22-4

The book is enjoyable, but it appears to give students false confidence. A good source of additional exercises.

Ref. 6 Glyn James 2001 Modern Engineering mathematics
3rd Edition Prentice Hall, 980 pp.
ISBN 0 130 18319 9

A very good book for students with an extra interest in maths

Ref. 7 A. Croft and R. Davidson Foundation Maths
4th Edition, Prentice Hall, 522 pp.
ISBN-13: 978-0-13-197921-5
ISBN-10: 0-13-197921-3

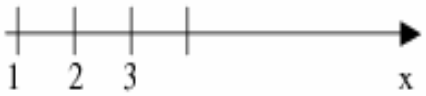
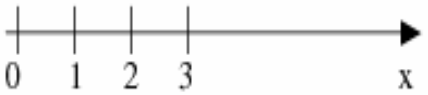
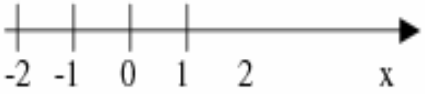
A good book for covering the pre-University foundations.

There are also many resources available now on internet. To start with you can consult http://www.lboro.ac.uk/research/helm/

Do not forget to use MATHLETICS for self-assessment! (The User Manual available on Blackboard).

HANDOUTS

ALGEBRA OF NUMBERS

OPERATIONS	NUMBERS
Addition (direct operation) 1. $a + b = b + a$ Terminology : a and b are called **terms** $a + b$ - **sum** 2. $(a + b) + c = a + (b + c)$ Conventions: $+ (b + c) = + b + c$ $+ a + b = a + b$	**Whole numbers** are 1, 2, 3, ...  addition of whole numbers gives a whole number
Subtraction (inverse operation) Def : $a - b = x$: $x + b = a$ (**: - such that**) Note: $a + b - b = a$ (subtraction undoes addition) 3. $a + 0 = a$ 4. for each a there exists one **additive inverse** -a : $a + (- a) = 0$ **Rules** $-(- a) = a$ $-(a) = -a$	Introduces **0** and **negative numbers**: $a-a=0$ if $b>a$ $a - b = - (b - a)$ **Natural numbers** are 0, 1, 2, ...  **Integers** are ..., -2, -1, 0, 1, 2, ...  **Absolute value (modulus)** of a $\|a\| = \begin{cases} a \text{ if } a \geq 0 \\ -a \text{ if } a > 0 \end{cases}$

Multiplication (direct operation)

Notations: $ab = a\cdot b = a\times b$
$2b = 2\cdot b = 2\times b$
$23 \neq 2\cdot 3,\ 23 = 2\cdot 10+3$
$2\frac{1}{2} \neq 2\cdot\frac{1}{2},\ 2\frac{1}{2} = 2+\frac{1}{2}$
$2\frac{3}{2} = 2\cdot\frac{3}{2}$

1. $a\cdot b = a\cdot b$
   Terminology: a and b are called **factors**
   ab - **product**
2. $(a\cdot b)\cdot c = a\cdot(b\cdot c)$
   Conventions: $abc = (ab)c$
   $a(-bc) = -abc$
3. $a(b+c) = ab+ac$
   $\longrightarrow$ Removing brackets
   $\longleftarrow$ factorising
4. $a\cdot 0 = 0$
5. $a\cdot 1 = a$

Rules:

$(a+b)(c+d) = ac+ad+bc+bd$ (**SMILE RULE**)
$(-1)n = -n$
$(-1)(-1) = 1$

Division (**inverse operation**)

Def: $a/b = x$ **:** $xb = a$

Terminology: a – numerator
b – denominator
a/b – fraction (ratio)
proper fraction if $|a|<|b|$, a, b integers

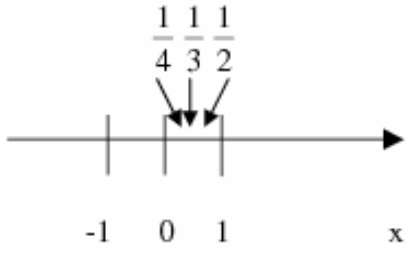
Note: $ab/b = a$ (division undoes multiplication)

6. For each $a\neq 0$ there exists one **multiplicative inverse** $1/a$ **:** $a\cdot 1/a = 1$

For whole numbers n
$a\cdot n = a+.....+a$
(n times)

Introduces **rational numbers**

Def: Rational numbers are all numbers $\frac{m}{n}$,
where m and $n\neq 0$ are integers
(**division by zero is not defined**)



Rules:

$$\frac{a}{b}\cdot n = \frac{an}{b}$$

$$\frac{a/b}{n} = \frac{a}{bn} = \frac{a/n}{b}$$

$$\frac{an}{bn} = \frac{a \cdot \cancel{n}}{b \cdot \cancel{n}} = \frac{a}{b}$$ **CANCELLATION**

$$\frac{\frac{1}{n}}{m} = \frac{m}{n}$$ **FLIP RULE**

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$$ Note: $\frac{a+c}{b}$ = (a+c)/b

$$\frac{a}{b}+\frac{c}{d} = \frac{ad}{bd}+\frac{cb}{db} = \frac{ad+cb}{bd}$$ **COMMON DENOMINATOR**

n-th power b^n (direct operation)

If n – a whole number

$b^n = \underbrace{b \cdot b \cdot b \cdot \ldots \cdot b}_{n \text{ times}}$

Rules

$a^m \cdot a^n = a^{m+n}$
(product of powers with the same base is a power with indices added)

$a^n \cdot b^n = (ab)^n$
(product of powers is power of product)

$a^m/a^n = a^{m-n}$
(ratio of powers is power of ratios)

$a^m/b^m = (a/b)^m$
(ratio of powers with the same base – subtract indices)

$a^0 = 1$

$a^{-n} = 1/a^n$

$(a^m)^n = a^{mn}$ [Convention: $a^{m^n} = a^{(m^n)}$]

n-th root (inverse to taking to power n)

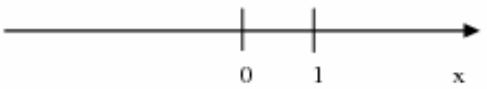
Introduces **irrrational** (not rational) **numbers**
(e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$; $\sqrt[3]{2}$, $\sqrt[3]{3}$, etc)

Def: $\sqrt[n]{b} = x : x^n = b$

Note: $\sqrt[n]{b^n} = b$
(taking n-th root undoes taking n-th power)

Therefore, can use notation $b^{1/n} = \sqrt[n]{b}$
($\sqrt[n]{b^n} = (b^n)^{1/n} = b^{n \cdot \frac{1}{n}} = b^1 = b$)

Real numbers are all rationals and all irrationals combined
Corresponding points cover the whole real line



Logarithm base b (inverse to taking to b power)

Introduces **irrrational** (not rational) **numbers**
(e.g. $\log_{10} 2$, $\log_{10} 3$, etc)

Def: $\log_b a = n$: $b^n = a$

Note: $\log_b b^n = n$ $(b^n = b^n)$

Roots and logs also introduce **imaginary** (not real) **numbers**
(e.g. $\sqrt{-1}, \log_{10}(-1)$ etc.)

Rules:

$\log_b xy = \log_b x + \log_b y$ (log of a product is sum of logs)
$\log_b x/y = \log_b x - \log_b y$ (log of a ratio is difference of logs)
$\log_b 1 = 0$ (log of 1 is 0)
$\log_b b = 1$ ($b^1 = b$)
$\log_b 1/a = -\log_b a$
$\log_b x^n = n \log_b x$ (log of a power is power times log)
$\log_b a = \log_c a/\log_c b$ (changing base)

ALGEBRA

General remarks

$a - b = a + (-b)$ ⟶ difference can be re-written as a sum

Addition is inverse to subtraction (just as subtraction is inverse to addition)

$\frac{a}{b} = a \cdot \frac{1}{b}$ ⟶ ratio can be re-written as a product

Multiplication is inverse to division (just as division is inverse to multiplication)

$\sqrt[n]{b} = b^{1/n}$ ⟶ root can be re-written as a power

Taking n-th power is inverse to taking n-th root (just as taking n-th root is inverse to taking n-th power)

$a = b^{\log_b a}$ ⟶ log is an exponent

Raising b to power is inverse to taking log base b (just as taking log b is inverse to raising b to power)

Order Of Operations

B(rackets): {[()]} (do inside out), P(owers) M(ultiplication), A(ddition)

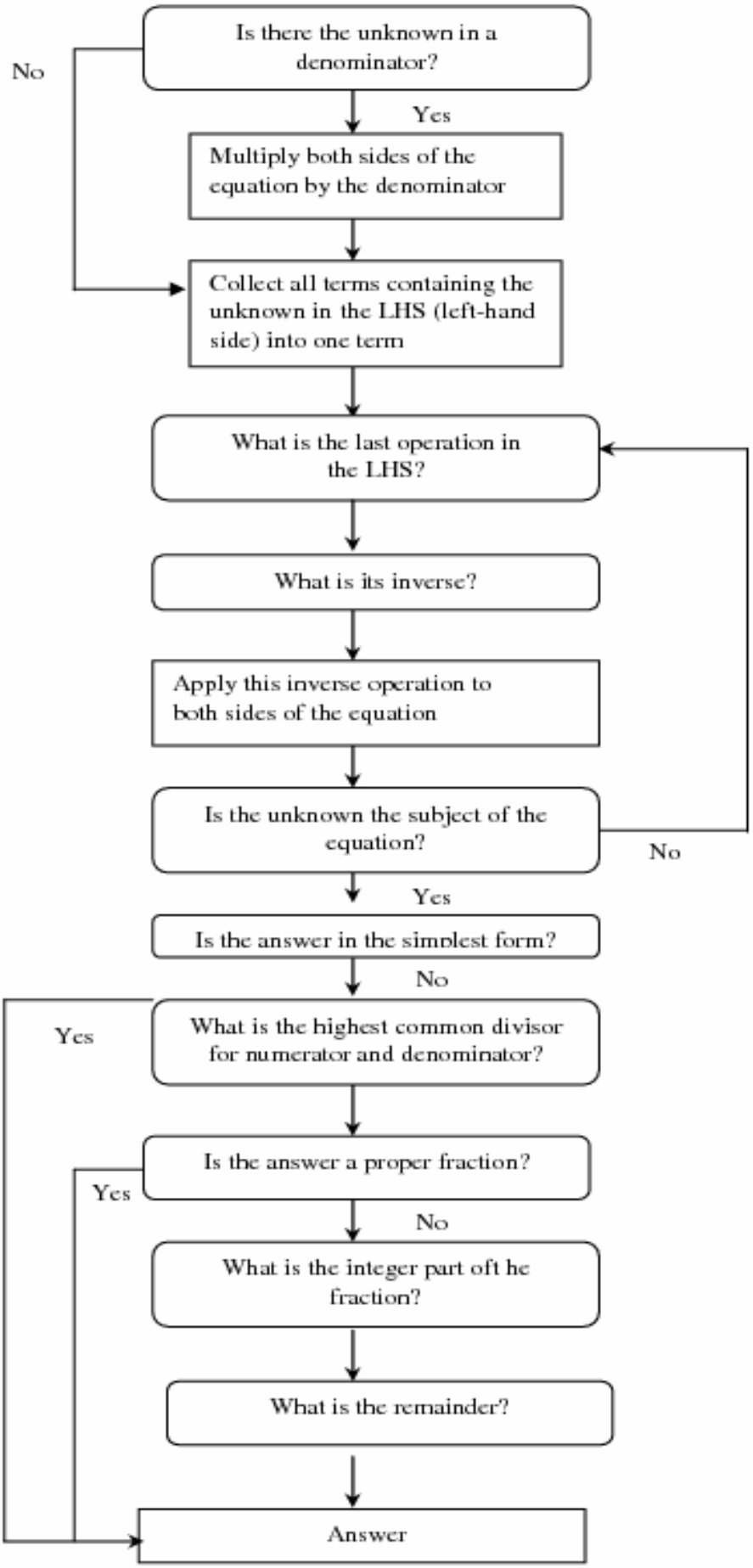
⟶

B	f( )	P	M	A
including		powers	×	+
implicit		roots	÷	-
		logs		

Brackets are not an operation, but they change the order of operations.

Any expression raised or lowered should be viewed as enclosed in invisible brackets.

DECISION TREE FOR SOLVING SIMPLE ALGEBRAIC EQUATIONS



FINDING A LIMIT OF A SEQUENCE

To find a limit of a sequence (as $n \to \infty$) use the first principles (graphical representation) or

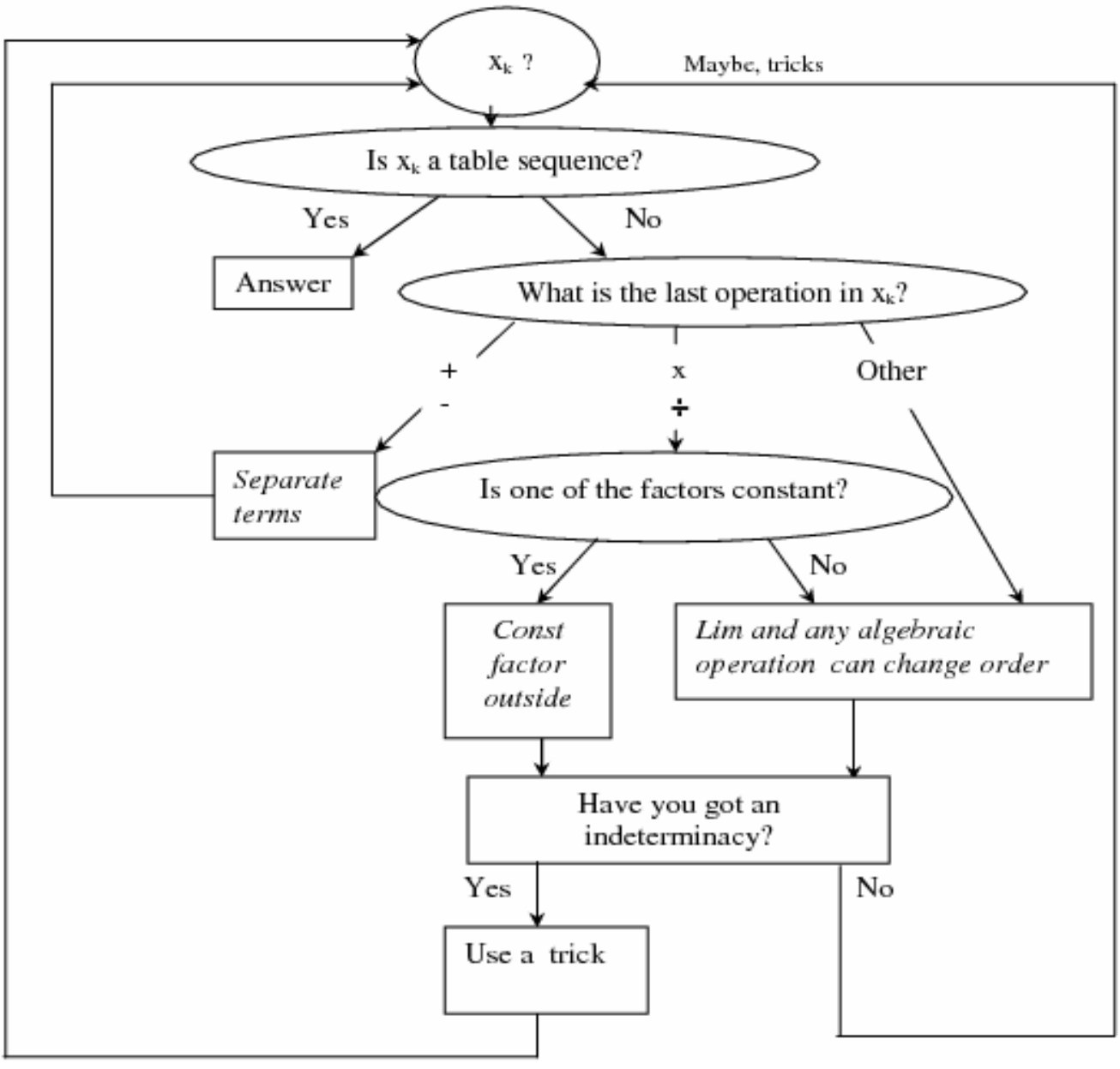
TABLE

x_k	$\lim x_k$
const	const
k	∞
$\frac{1}{k}$	0
$a^{\frac{1}{n}}$ (a>0)	1
q^n (|q|<1)	0

RULES

1. $\lim \alpha x_k = \alpha \lim x_k$ – *const factor ousidet*
2. $\lim [x_k \pm y_k] = \lim x_k \pm \lim y_k$ – *separate terms*
3. $\lim (x_k\, y_k) = \lim x_k \lim y_k$ – *product rule*
4. $\lim \frac{x_k}{y_k} = \frac{\lim x_k}{\lim y_k}$ – *quotient rule*
5. $\lim f(x_k) = f(\lim x_k)$ – *limit and any algebraic operation or simple function can change places (commute)*

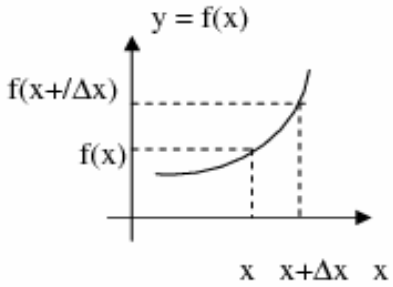
DECISION TREE



DIFFERENTIATION

To differentiate a function use the first principles

Def: $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$



or represent roots as powers and use

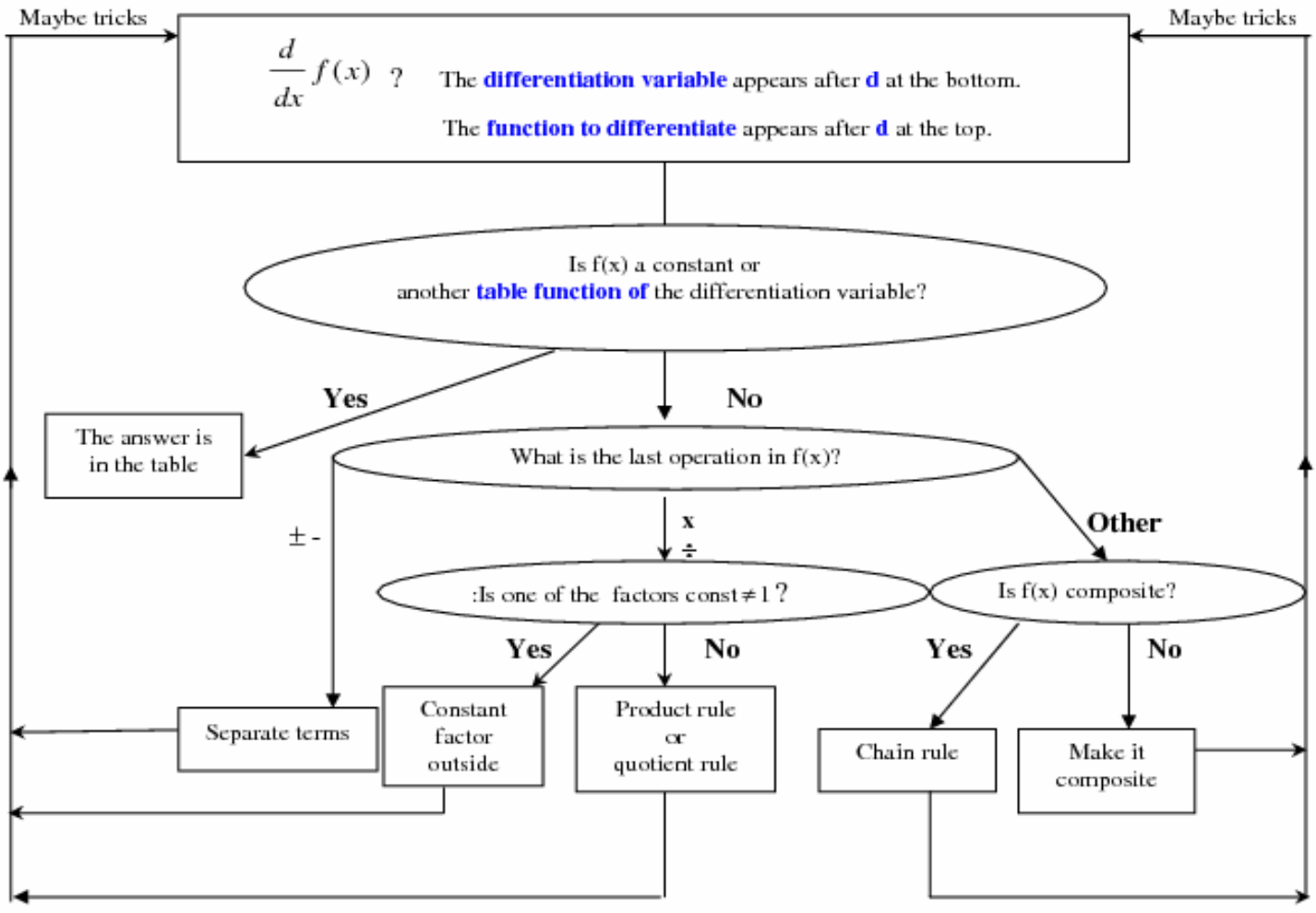
TABLE

f (x)	$\frac{d}{dx} f(x)$
const	0
x^n	nx^{n-1}
e^x	e^x
ln x	$\frac{1}{x}$
sin x	cos x
cos x	-sin x

RULES

1. $\frac{d}{dx}[\alpha f(x)] = \alpha \frac{df(x)}{dx}$ - *const factor ousidet*
2. $\frac{d}{dx}[f(x) + g(x)] = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$ – *separate terms*
3. $\frac{d}{dx}[f(x)g(x)] = \frac{df(x)}{dx} g(x) + \frac{dg(x)}{dx} f(x)$ – *product rule*
4. $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{\frac{df}{dx} g(x) - \frac{dg}{dx} f(x)}{g^2}$ – *quotient rule*
5. $\frac{d}{dx}[f(g(x)] = \frac{df}{dg}\frac{dg}{dx}$ – *chain rule* (decompose, differentiate, multiply)

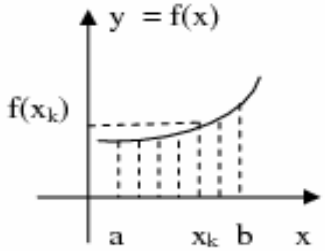
DECISION TREE



INTEGRATION

To find a definite integral of a function use the definition

$$\text{Def: } \int_a^b f(x)dx = \lim_{\Delta x \to 0} \sum_{x_k=a}^{b} f(x)\Delta x$$



or represent roots as powers and use

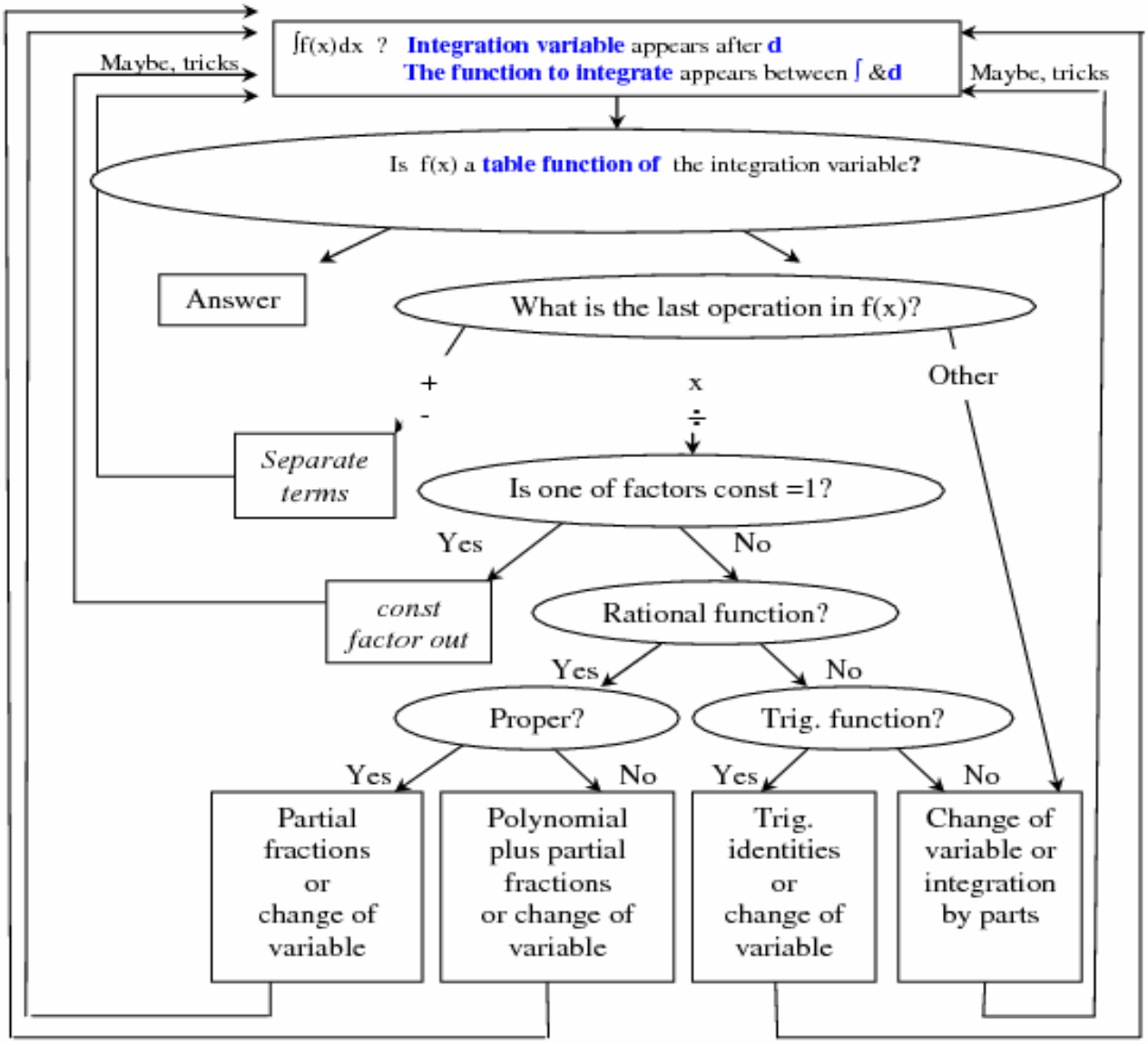
TABLE

f(x)	$\int f(x)dx$
x^n, $n \neq -1$	$\frac{x^{n+1}}{n+1}+c$
1/x	$\log \lvert x \rvert + c$
e^x	$e^x + c$
sin x	- cos x + c
cos x	sin x + c

RULES

1. $\int \alpha f(x)dx = \alpha \int f(x)dx$ - *const factor outside*
2. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ - *separate terms*

DECISION TREE

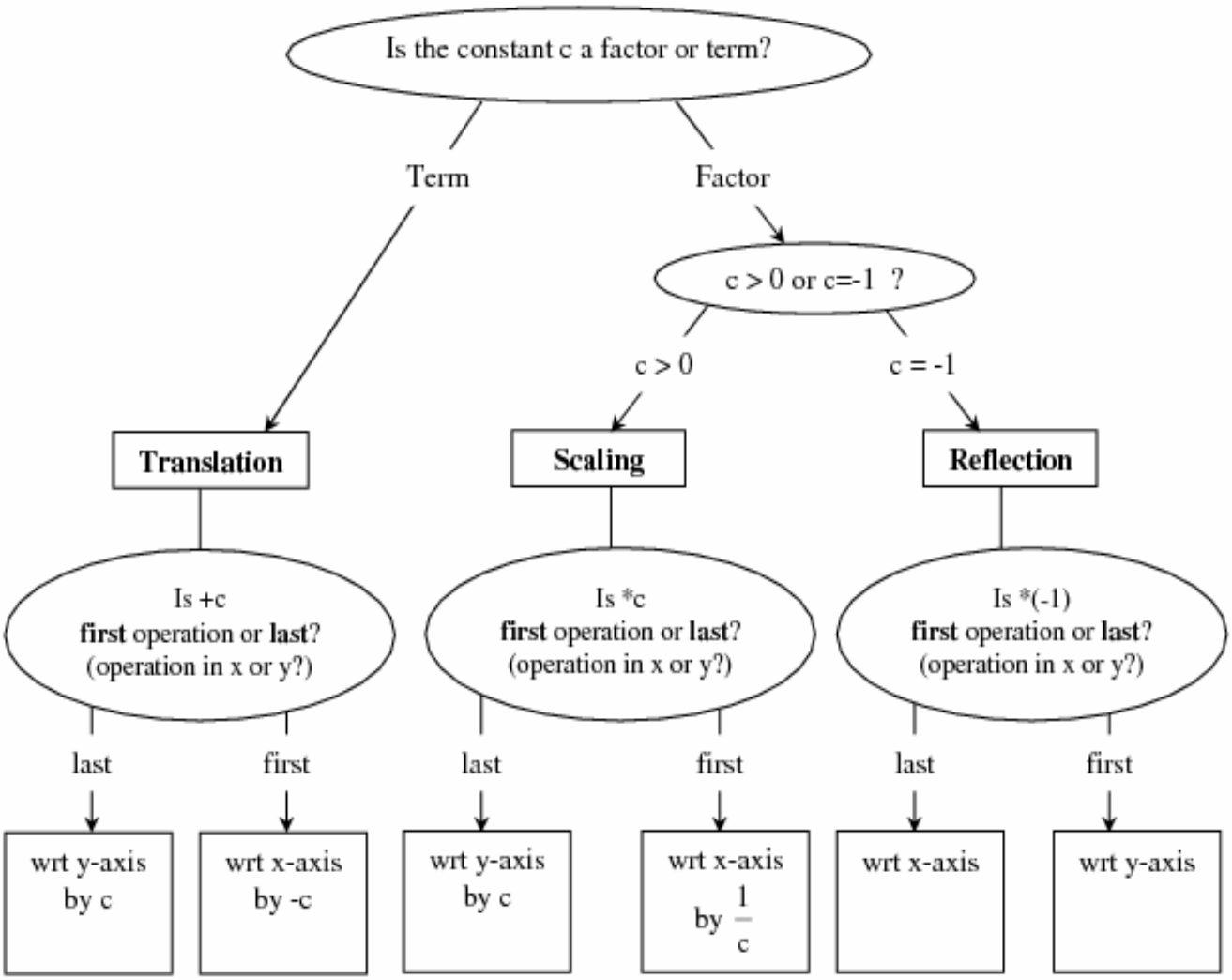


SKETCHING GRAPHS BY SIMPLE TRANSFORMATIONS OF INDEPENDENT OR DEPENDENT VARIABLE

Note 1: if c is a **negative factor**, write $c = (-1) * |c|$, so that there are two simple transformations.

Note 2: if c is a term which affects **neither first operation nor last**, the Decision Tree is not applicable. Try algebraic manipulations.

Decision Tree for Sketching by Simple Transformations of y=f(x), which are due to multiplication by, or addition of, a constant c



Note 3: If **the same operation** is applied to x in all positions in the equation this operation still should be treated as the first operation, as in $ln\,(x-2)+sin(x-2)$.

Note 4: if y is given **implicitly rather than explicitly**, so that the equation looks like $f(x,y)=0$, then in order to see what transformation is effected by adding a constant c to y or multiplying it by a constant c, y should be made the subject of the functional equation. Therefore, similarly to transformations of x above, the corresponding transformation of y is defined by the inverse operation, -c or $\frac{1}{c}$, respectively.

ORDER OF OPERATIONS

When EVALUATING a mathematical EXPRESSION it is important to know the order in which the OPERATIONS must be performed. The ORDER OF OPERATIONS is as follows:

First, expression in **BRACKETS** must be EVALUATED. If there are several sets of brackets, e.g. {[( )]}, expressions inside the inner brackets must be EVALUATED first. The rule applies not only to brackets explicitly present, but also to brackets, which are implied. Two special cases to watch for are fractions and functions. Indeed, when a fraction (a+b)/(c+d) is presented as a two-storey fraction the brackets are absent, and some authors do not bracket ARGUMENTS of elementary FUNCTIONS, such as exp, log, sin, cos, tan *etc*. In other words, e^x should be understood as exp(x), sin x as sin(x) *etc*.

Other OPERATIONS must be performed in the order of decreasing complexity, which is
FUNCTIONS f ( )
POWERS (including inverse operations of roots and logs)
MULTIPLICATION (including inverse operation of division)
ADDITION (including inverse operation of subtraction)

That is, the more complicated OPERATIONS take precedence. For simplicity, we refer to this order by the abbreviation **BFPMA**.